

Solar Radiation and Asteroidal Motion

Letter to the Editor

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Abstract. Effects of solar wind and solar electromagnetic radiation on motion of asteroids are discussed. The results complete the statements presented in Vokrouhlický and Milani (2000).

As for the effect of electromagnetic radiation, the complete equation of motion is presented to the first order in v/c – the shape of asteroid (spherical body is explicitly presented) and surface distribution of albedo should be taken into account. Optical quantities must be calculated in proper frame of reference.

Key words: celestial mechanics, stellar dynamics, asteroids, interplanetary dust

1. Introduction

Vokrouhlický and Milani (2000) investigate motion of asteroids due to direct solar radiation pressure. They have come to the conclusion that Poynting-Robertson effect (P-R effect; Robertson 1937) should be also considered.

The aim of this paper is to call attention towards i) the significance of the solar wind in comparison with the P-R effect, and, ii) the possible significance of the real equation of motion originating from the interaction of solar electromagnetic radiation with an asteroid.

As for the effect of the solar wind, we use the equation of motion derived in Klačka (1994), see also Klačka (1999a). As a consequence, application to 1566 Icarus, the effect of solar wind yields, e. g., that the current secular decrease of semi-major axis is in 0.20 larger than for the P-R effect for perfect absorption; the value 0.20 is a little larger than the value $4a_0/9$ used in Vokrouhlický and Milani (2000).

3. Solar Electromagnetic Radiation

Taking into account Eqs. (6), (7) and (29) in Klačka (2000c) we can write equation of motion for spherical body due to its interaction with electromagnetic radiation:

$$\begin{aligned} \frac{d\mathbf{v}}{dt} &= \frac{S}{m c} \pi R^2 \left\{ \left(1 - \frac{\mathbf{v} \cdot \hat{\mathbf{S}}_i}{c} \right) \hat{\mathbf{S}}_i - \frac{\mathbf{v}}{c} \right\} + \frac{S}{m c} \frac{2}{3} R^2 X, \\ X &= \left\{ \left(1 - \frac{\mathbf{v} \cdot \hat{\mathbf{S}}_i}{c} \right) \hat{\mathbf{S}}_i - \frac{\mathbf{v}}{c} \right\} \int_0^{2\pi} d\varphi' \int_0^{\pi/2} d\vartheta' a'_0(\vartheta', \varphi') \sin\vartheta' \cos^2\vartheta' - \\ &\quad - \left\{ \left(1 - 2 \frac{\mathbf{v} \cdot \hat{\mathbf{S}}_i}{c} + \frac{\mathbf{v} \cdot \hat{\mathbf{e}}_1}{c} \right) \hat{\mathbf{e}}_1 - \frac{\mathbf{v}}{c} \right\} \times \\ &\quad \times \int_0^{2\pi} d\varphi' \int_0^{\pi/2} d\vartheta' a'_0(\vartheta', \varphi') \sin^2\vartheta' \cos\vartheta' \cos\varphi' - \\ &\quad - \left\{ \left(1 - 2 \frac{\mathbf{v} \cdot \hat{\mathbf{S}}_i}{c} + \frac{\mathbf{v} \cdot \hat{\mathbf{e}}_2}{c} \right) \hat{\mathbf{e}}_2 - \frac{\mathbf{v}}{c} \right\} \times \\ &\quad \int_0^{2\pi} d\varphi' \int_0^{\pi/2} d\vartheta' a'_0(\vartheta', \varphi') \sin^2\vartheta' \cos\vartheta' \sin\varphi', \end{aligned} \quad (1)$$

where $a'_0(\vartheta', \varphi')$ is surface albedo of a spherical body (asteroid) of mass m and radius R moving around the Sun with orbital velocity \mathbf{v} , c is the velocity of light, S is the flux density of the solar electromagnetic radiation, $\hat{\mathbf{S}}_i$ is unit vector of the incident radiation, unit vectors $\hat{\mathbf{e}}_1 = (1 - \mathbf{v} \cdot \hat{\mathbf{e}}'_1/c) \hat{\mathbf{e}}'_1 + \mathbf{v}/c$, $\hat{\mathbf{e}}_2 = (1 - \mathbf{v} \cdot \hat{\mathbf{e}}'_2/c) \hat{\mathbf{e}}'_2 + \mathbf{v}/c$ (analogous relation holds for $\hat{\mathbf{S}}_i$), $\hat{\mathbf{e}}'_1 \cdot \hat{\mathbf{e}}'_2 = 0$, $\hat{\mathbf{e}}'_1 \times \hat{\mathbf{e}}'_2 = \hat{\mathbf{S}}'_i$. $\varphi' = 0$ for the direction and orientation $\hat{\mathbf{e}}'_1$, ϑ' is measured from $-\hat{\mathbf{S}}'_i$, both angles are measured in positive directions. The numerical factor $2/3$ comes from the considered model – diffuse reflection corresponding to the Lambert's law (although it may not hold for real bodies in Solar System – see p. 112 in Van de Hulst (1957)): $\int_2 \pi \{(\cos\alpha)/\pi\} \cos\alpha d\Omega = 2/3$, $d\Omega = 2\pi \sin\alpha d\alpha$.

If albedo a'_0 is a constant for the whole sphere, then Eq. (1) reduces to

$$\frac{d\mathbf{v}}{dt} = \frac{S}{m c} \pi R^2 \left(1 + \frac{4}{9} a'_0 \right) \left\{ \left(1 - \frac{\mathbf{v} \cdot \hat{\mathbf{S}}_i}{c} \right) \hat{\mathbf{S}}_i - \frac{\mathbf{v}}{c} \right\}, \quad (2)$$

which is the P-R effect to the first order in v/c for our case ($Q'_{PR} = 1 + 4a'_0/9$ – see Eq. (122) in Klačka 1992a).

Eq. (1) represents equation of motion for interaction between spherical particle and electromagnetic radiation (thermal reemission of the absorbed energy connected with thermal conductivity of the surface material is not considered). As it was already stressed a special form of Eq. (1) is Eq. (2) which is the P-R effect.

Eq. (1) is real relativistic equation of motion to the first order in v/c . The access of Vokrouhlický and Milani (2000) corresponds to putting together some of its individual terms in a heuristic manner. They seem to be in coincidence with Eq. (1) if we neglect less significant terms in Eq. (1). However, our access is physically more correct since it shows that no other terms are important; moreover, some new terms of the order of v/c are presented. The important fact which one must bear in mind is that values of integrals in Eq. (1) correspond to proper (local) inertial frame of reference.

P-R effect is not the acceleration which is presented in Vokrouhlický and Milani (2000), as it is immediately seen from the comparison of our Eq. (2) and their Eq. (21). As for the correct equation for secular changes of orbital elements, together with initial conditions, we refer the reader to Klačka (1992b) and Klačka and Kaufmannová (1992). Vokrouhlický and Milani (2000) refer to Breiter and Jackson (1998). However, except confirming the results of Wyatt and Whipple (1950), (see Klačka (1992b), for more details; initial conditions are also important) and Klačka and Kaufmannová (1992) the paper by Breiter and Jackson (1998) yields only nonphysical results (nice analytical solution yielding still increasing eccentricity does not occur in reality due to higher orders in v/c in the P-R effect), as it is presented in Klačka (1999b). As for correct understanding of the P-R effect, we refer also to Klačka (2000b). As an soluble example for secular changes of orbital elements for nonspherical particle we refer to Klačka (2000a). As for relativistic covariant equation of motion we refer to Klačka (2000c).

4. Conclusion

We have shown the way in which the effect of solar wind must be taken into account in dealing with the motion of a body (asteroid).

We have presented complete equation of motion (to the first order in v/c) for spherical body under interaction with the electromagnetic radiation. The important statement concerns the fact that optical quantities must be calculated in the proper frame of reference of the body.

We have called attention towards the correct physics as for the P-R effect and solution of the corresponding equation of motion.

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